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**Final Progress Report on Research Supported by  
Grant AFOSR-88-0245.**

**Principal Investigator:** Professor Edward K. Blum, Mathematics  
Department, University of Southern California, Los Angeles, CA  
90089-1113.

**Title of Project:** Mathematical and Numerical Analysis Aspects of  
Quasi-Neural Networks.

Significant results were obtained on feed-forward and recurrent  
networks. Five technical papers were accepted for publication in  
leading journals.

1. Feed-forward networks. The P.I. and his Ph.D. student,  
Leong Li, obtained a basic result on feed-forward networks. This is  
described in the paper "Approximation theory and feed-forward  
networks", which has been published in the journal, *Neural Networks*  
(1991) 4, 511-515. The basic result is that feed-forward networks  
of McCulloch-Pitts neurons yield arbitrarily close piecewise-constant  
approximations to continuous and  $L_2$ -functions. The fundamental  
idea of piecewise-constant approximation was not recognized by  
previous researchers. By using this idea, Blum and Li were able to  
give a simple technique for constructing 3-layer networks to  
uniformly approximate arbitrary real continuous functions of  $n$   
variables and also arbitrary  $L_2$ -functions in the mean square norm  
with arbitrary accuracy. This technique should be useful in many of  
the applications of feed-forward networks. Li has also published a  
second paper in this topic.

2. Dynamical behavior of recurrent neural networks.  
Recurrent neural networks, that is, networks with feedback loops,  
exhibit all the phenomena found in classical dynamical systems  
governed by systems of ordinary differential equations (ODE's). In  
fact, if the neuron activities are modeled by ODE's, then the network  
is modeled as a system of coupled ODE's. However, it is also of



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interest to model the neurons as analog elements which have states (outputs) in the real interval  $I=[0,1]$  governed by an input-output function of sigmoidal type and operating in discrete-time. If the operation is synchronous, then the network is modeled as a system of difference equations rather than differential equations. Thus, the dynamics is that of iteration of a map  $F : I^m \rightarrow I^m$ , according to the general equation  $x(t+1)=F(x(t))$ ,  $t=0,1,2,\dots$ , where  $x(t)$  is the state vector at time  $t$ . In many applications (e.g., associative memories, oscillators of various kinds, etc.),  $F$  has the form  $F(x) = \sigma(Wx + L)$ , where  $W$  is a real "weight" matrix,  $L$  is the "threshold" vector and  $\sigma$  is a sigmoidal-type nonlinear vector function. In these applications, one is interested in the existence and stability of fixed-points and periodic orbits in phase space (i.e. oscillations). There is a natural parameter in such networks, namely, the "gains" of the neurons, as represented by the maximum slopes,  $a_i$ , of the sigmoidal functions. One is also interested in the changes in the dynamics induced by variations in the gains, in particular, in the occurrence of bifurcation points at which stable equilibria become unstable or at which stable oscillations appear and disappear.

The P.I. and his Ph.D. student, Xin Wang, have studied the dynamics of such networks and obtained a number of new results which should be useful in applications. These are described in detail in the paper "Stability of Fixed Points and Periodic Orbits and Bifurcations in Analog Neural Networks", accepted for publication in the journal, *Neural Networks*.

The case of symmetric  $W$  has received attention in the recent literature. We consider the simplest instance of such a network having only 2 neurons coupled to operate like an analog multivibrator circuit. We show that the dynamics of fixed points and periodic orbits (oscillations) can be completely characterized by elementary analysis without introducing a Liapunov function, the usual tool. This analysis can be extended to special networks built from these 2-neuron modules. The same methods of analysis are then applied to non-symmetric  $W$ . For example, it is shown how to

construct ring oscillators of any period. The case of coupled oscillators and forced oscillators is also subjected to the same analysis. By varying the gains of the sigmoidal neurons we obtain interesting bifurcations into oscillatory behavior.

3. Chaotic dynamics in neural networks. It is known that neural networks are capable of chaotic behavior. New results are reported in the paper, "Period-doublings to Chaos in a Simple Neural Network: A Mathematical Proof" by Xin Wang (Ph.D. student of the P.I.). Usually, chaotic dynamics is demonstrated by simulation. In this part of the project, Wang was able to prove that chaos occurs in a 2-neuron network with self-loops. Thus, this simple network differs from the 2-neuron network studied by Blum and Wang described above. However, the dynamics is again discrete-time iteration of a nonlinear sigmoidal map. By restricting the weight matrix,  $W$  to be singular, it is possible to reduce the dynamics to iteration of a family of 1-dimensional maps parametrized by the gain,  $\mu$  of the neurons. Appealing to the known theory of 1-dimensional maps, Wang is able to prove that the 2-neuron networks with singular  $W$  follow the period-doubling route to chaos as  $\mu$  increases. To illustrate the theoretical result, Wang computes the bifurcation diagrams for two networks with different  $W$ 's and demonstrates the occurrence of chaos as  $\mu$  increases.

4. Discrete-time versus continuous-time dynamics. Blum and Wang compare the dynamical properties of continuous-time neural networks defined by systems of differential equations of the leaky integrator type with the dynamical properties of the approximating difference equations. Quantitative results in the step-size of the discrete-time numerical method to guarantee the same dynamics are obtained.

### Publications:

1. Blum, E.K. (1989). Approximation of Boolean functions by sigmoidal networks, *Neural Computation* 1, 532-540.
2. Blum, E.K. (1990). Mathematical aspects of outer-product asynchronous content-addressable memories, *Bio. Cyber.* 62, 337-348.
3. Blum, E.K. and Li, L.K. (1991). Approximation theory and feed-forward networks, *Neural Networks* 4, 511-515.
4. Blum, E.K. and Wang, X. (1992). Stability of fixed-points and periodic orbits and bifurcations in analog neural networks, *Neural Networks* (In press).
5. Blum, E.K. and Wang, X. (1992). Discrete-time versus continuous-time models of neural networks, *J. Computer & System Sciences*, (In press).
6. Wang, X. (1992). Periodic-doublings to chaos in a simple neural network: a mathematical proof, *Complex Systems*.